Non Local Heat Flux in Weakly Collisional Plasmas

Juan R. Sanmartín
E.T.S.I. Aeronáuticos, Universidad Politécnica
28040 Madrid, Spain
Fax: (34-1) 3366303
Email: jrs@faia.upm.es

Standard transport theory is applicable when the characteristic macroscopic length, H, is larger than the mean free path for collisions that lead to a Maxwellian distribution. In a mixture of two neutral gases, temperatures and mean directed velocities are nearly the same for either species, the small relative velocity between them being important only in the sense that it will induce changes of concentration throughout the system, that is, for diffusion. Naturally, particle number densities may be arbitrarily different.

A plasma behaves just the opposite in this respect. The Debye length being short when compared with all mean free paths, transport processes are basically quasi-neutral, the density ratio being a constant, $n_e/n_i \simeq$ ion charge state Z. On the other hand, and because the electron-to-ion mass ratio m_e/m_i is very small, energy equipartition need not be established in a distance H, hence, for generality, one should allow different temperatures, i.e., $T_e \neq T_i$, the use of a single one unduly restricting the analysis. Also, the difference between ion an electron velocity, $\bar{u} \equiv \bar{u}_e - \bar{u}_i$, may be as large as $\bar{u}_e$ or $\bar{u}_i$. For instance, in an expansion into vacuum, as in laser fusion, characteristic velocities are of the order $(ZT_e/m_i)^{1/2}$. If λ_T is the mean free path for scattering of thermal electrons by ions, the friction between species requires that u be, at most, of the order of $(T_e/m_e)^{1/2}\lambda_T/H$, and therefore comparable, in principle, to u_i or u_e.

In a monatomic gas, both viscous and thermal diffusivities are roughly given by the product of mean free path and thermal speed, their ratio, the Prandtl number, is near unity. A mixture of such gases has an effective Prandtl number around unity, also. A plasma is again quite different in this respect. Ion diffusivities are smaller than electron diffusivities by a factor of order $(m_e/m_i)^{1/2}$. In addition, since electron momentum convection is itself negligible, one may also ignore the electron viscous tensor. The effective Prandtl number of a plasma is therefore very low, electron heat conduction being the dominant diffusion process. In fact, the structure, for instance, of a plasma shock may be entirely determined by heat flow, when viscosity does count, its effects, due to ions, are limited to a thin sublayer within the overall shock structure. The case of the coronal plasma ablated from a laser target is, in this sense, quite similar.

In laser fusion, H is often much larger than λ_T: for a characteristic speed $u_e \sim u_i \sim (ZT_e/m_i)^{1/2}$, and from a balance of conduction and convection in the overdense plasma, $u_e \sim \lambda_T(T_e/m_e)^{1/2}/H$, we have

$$H/\lambda_T \sim (m_i/Zm_e)^{1/2} \sim 60$$

One might thus expect classical transport results to generally hold in the plasma blowing off a laser target. In the last 20 years, however, there has been experimental evidence showing that classical results fail at such large H/λ_T, a fact that requires an explanation.

The strong energy dependence of plasma mean free paths, characteristic of a Coulomb cross section, might explain this fact through a failure of classical transport theory that appears when main-body electrons are still highly collisional. For strongly collisional conditions, the electron distribution function takes the usual form,

$$f_e(\bar{w}) = f_M(w)[1 + \varphi(\bar{w})], \qquad |\varphi| \ll 1,$$

where f_M is a Maxwellian distribution. If Z is large (so that ion-electron collisions dominate the collision term in the electron kinetic equation), and in absence of external magnetic field and relative velocity $\bar{u}$, φ takes the simple form

$$\varphi = -\tau_{ei}(\tilde{\varepsilon} - 4)\bar{w} \cdot \nabla \ln T_e$$

where $\tilde{\varepsilon} \equiv m_e w^2/2T_e$ and $\tau_{ei} \propto w^3$ is a characteristic ion-electron collision time at electron velocity w. The integral for the heat flux now becomes

$$q_T \propto \int_0^\infty \tilde{\varepsilon}^4(\tilde{\varepsilon} - 4)e^{-\tilde{\varepsilon}} d\tilde{\varepsilon}.$$

The integrand has a maximum at $\tilde{\varepsilon} = \tilde{\varepsilon}^* \simeq 6.5$, so that electrons contributing most to the heat flux lie in the tail of the distribution function. Since $w\tau_{ei} \propto \varepsilon^2$, one could have both φ small at thermal energies, $\tilde{\varepsilon} \sim 1$, and $\varphi \sim 1$ at the energies of interest, $\tilde{\varepsilon} \sim \tilde{\varepsilon}^*$.

A self-consistent transport model at large Z, when f_e may fail to be Maxwellian while still being isotropic, was proposed by Albritton et al.[1] For $\tilde{\varepsilon}^*$ electrons we write $f_e(\bar{w}) = f_0(w)[1 + \varphi(\bar{w})]$, φ small, with $f_0 \neq f_M$ in general; at thermal energies, we will have $f_0 = f_M$. Consider the electron kinetic equation, neglecting ion-collision effects of order m_e/m_i,

$$\bar{w} \cdot \nabla f_e - \frac{e\bar{E}}{m_e} \cdot \frac{\partial f_e}{\partial \bar{w}} = C_{ee}(\bar{w}) + C_{ei}(\bar{w}) \simeq C_{ee} + \frac{\partial}{\partial \bar{w}} \cdot \frac{\bar{\bar{I}}\, w^2 - \bar{w}\bar{w}}{2\tau_{ei}} \cdot \frac{\partial f_e}{\partial \bar{w}} \tag{1}$$

where C_{ei} represents pure scattering. To dominant terms we may set $f_e = f_0$ in the left-hand side of (1), and drop $C_{ee} \sim C_{ei}/Z$, to obtain

$$\bar{w} \cdot \left(\nabla f_0 - \frac{e\bar{E}}{m_e w} \frac{\partial f_0}{\partial w} \right) = C_{ei} \ (f_0 \varphi).$$

Since f_0 is yet unknown, it is worth simplifying this equation by writing $f_0(\bar{r}, w)$ as $f_0(\bar{r}, \varepsilon \equiv \frac{1}{2}\, m_e w^2 - e\Psi)$, with $\bar{E} \equiv -\nabla\Psi$:

$$\bar{w} \cdot \nabla f_0 = C_{ei} \ (f_0 \varphi).$$

Now the local Maxwellian takes the form $f_M \equiv n(m_e/2\pi T_e)^{3/2}\exp[-(\varepsilon + e\Psi)/T_e]$, $n_e \equiv n$. Trying $\varphi = \bar{w} \cdot \bar{g}(w)/f_0$ we find $C_{ei} = -\bar{w} \cdot \bar{g}/\tau_{ei}$ and

$$\bar{g} = -\tau_{ei} \nabla f_0. \tag{2}$$

If $f_0 \to f_M$, use of $\bar{g}$ recovers Spitzer's value for $\bar{q}$.

To determine f_0, we average (1) over velocity angles; f_0 on the left, and C_{ei}, give no contribution. We find, using (2)

$$-\frac{1}{3} w \nabla \cdot (\tau_{ei} w \nabla f_0) = C_{ee}(f_0, f_0) \tag{3}$$

Since we will have $e\Psi \sim T_e$, we have omitted a term $(2\tau_{ei} e\bar{E}/3m_e) \cdot \nabla f_0$, which is small by a factor $2e\Psi/m_e w^2 \sim 1/\tilde{\varepsilon}^*$, or 15%; note that the collision term itself has only logarithmic accuracy. In handling (3) we may similarly set $\frac{1}{2}m_e w^2 \equiv \varepsilon + e\Psi \simeq \varepsilon$, when appearing in powers (but not in an exponential like the Maxwellian!).

The self-collision term C_{ee} may be approximated by using the fact that thermal electrons, which are Maxwellian, dominate collisions with ε^*-electrons,

$$C_{ee} \simeq \frac{m_e w^2}{\tau_{ei} Z} \frac{\partial}{\partial \varepsilon}\left(f_0 + T_e \frac{\partial f_0}{\partial \varepsilon}\right).$$

Since T_e/ε is small, the last term should be neglected, for consistency, if f_0 followed a power law, but not if it were an exponential like the Maxwellian; actually, C_{ee} above will vanish for $f_0 = f_M$. Here, one makes a crucial ansatz:

$$|T_e \partial(f_0 - f_M)/\partial\varepsilon| \ll |f_0 - f_M| \tag{4}$$

and thus obtains

$$C_{ee} = \frac{m_e w^2}{Z\tau_{ei}}\left(\frac{\partial f_0}{\partial \varepsilon} - \frac{\partial f_M}{\partial \varepsilon}\right). \tag{5}$$

Using (5) in (3), taking the heat flux along x and defining $\mathrm{d}\xi \equiv (3/8Z_i)^{1/2} m_e^2 w^3 \tau_{ei}^{-1} \mathrm{d}x$, we arrive at an equation for f_0,

$$\frac{\partial f_0}{\partial \varepsilon} + \varepsilon^3 \frac{\partial^2 f_0}{\partial \xi^2} = \frac{\partial f_M}{\partial \varepsilon}. \tag{6}$$

This is a " heat diffusion" equation with $-\varepsilon^4/4$ as a time-like variable. For an infinite plasma, with f_0 vanishing in the " remote past" ($\varepsilon \to \infty$), the solution is clearly [1]

$$f_o(\xi,\varepsilon) = \int_{-\infty}^{\infty} \frac{d\xi'}{\pi^{1/2}} \int_{\varepsilon}^{\infty} \frac{f_M(\xi',\varepsilon')\mathrm{d}\varepsilon'}{T_e'(\varepsilon'^4 - \varepsilon^4)^{1/2}} \exp\left[\frac{-(\xi-\xi')^2}{\varepsilon'^4 - \varepsilon^4}\right] \tag{7}$$

which can be shown to satisfy the ansatz; here $T_e' \equiv T_e(\xi')$. Note that the non-Maxwellian population for suprathermal energy ε at a position ξ arises from a *Maxwellian source of electrons at* $\varepsilon' > \varepsilon$, which lost energy while random walking from a neighbor position ξ'!

For a profile with comparable high and low temperatures T_h, T_l, or if $T_e \sim T_h \gg T_l$, we will have $\varepsilon \gg T_e'$. Introducing a second ansatz, $H d\xi/dx \ll \varepsilon^{5/2}/T_e'^{1/2}$, only values of ε' close to ε are found to contribute to the ε'-integral, which can then be carried out: [2]

$$f_o(\xi,\varepsilon) = \int_{-\infty}^{\infty} \frac{f_M(\xi',\varepsilon)\mathrm{d}\xi'}{2(\varepsilon^3 T_e')^{1/2}} \exp\left[\frac{-|\xi-\xi'|}{(\varepsilon^3 T_e')^{1/2}}\right]. \tag{8}$$

The parameter range of interest is clearly $\Delta\xi \sim 2\varepsilon^{3/2} T_e'^{1/2}$, which is equivalent to

$$H \sim Z^{1/2} \bar{\varepsilon}^{*3/2} \lambda_T \tag{9}$$

The second ansatz follows immediately from (9) and $T_e' \ll \varepsilon$. Note also that the range in Eq.(9) agrees numerically with the characteristic distance between the ablation and critical surfaces, $H \sim (m_i/Z\, m_e)^{1/2}\lambda_T$, previously found. Again, for $T_e \sim T_l \ll T_h$ and (9) satisfied at the top of the profile, electrons carrying the heat from the top should be fully collisional at the typical bottom density $n_l \sim n_h T_h/T_l$:

For thermal energies, $\varepsilon \sim T_e$, and H satisfying (9), Eq.(8) is a convolution of $f_M(\xi', \varepsilon)$ and a δ-function, giving $f_0 \simeq f_M$. For a larger scale length, (8) would yield $f_0 \simeq f_M$ at the suprathermal energies of interest, $\varepsilon \sim 6.5\ T_e$, recovering the classical result. At the other extreme, when H is much less than the value in (9), Eq. (4) should ultimately break down. Since the model allows $f_0 - f_M$ to be comparable to f_M, Eq. (4) may be rewritten as

$$|\partial(f_0 - f_M)/\partial\varepsilon| \ll |\partial f_M/\partial\varepsilon|; \tag{10}$$

this is compatible with $|f_0 - f_M| \sim f_M$ only for a narrow range $\Delta\varepsilon \sim T_e$ (in the tail of the distribution) which, nonetheless, can include the electrons carrying most of the flux. If f_M changes in a "distance" $\Delta\xi \ll (\varepsilon^3 T_e)^{1/2}$ the solution to (6) lags well behind, and (10) will fail. Prasad and Kershaw [3] have illustrated the failure of (6) for some extreme and peculiar profiles. Ramirez [4] has used the model with full hydrodynamics to study the flow off a target and found it valid up to a laser intensity one order of magnitude above that for which classical transport fails. Unfortunately, the model will not signal its failure when used in a code.

Introducing (8) in the expressions for the particle and heat flux yields two coupled equations for q_T and an auxiliary field E_{nl}:

$$\begin{aligned} \{0, q_T\} &= \int \{1, \frac{1}{2} m_e w^2\} w_x d\bar{w}(-\tau_{ei}\bar{w} \cdot \nabla f_0) \\ &= \int \frac{-\{1, T_e'\} n' dx'}{4\pi(3m_e Z T_e')^{1/2}} \left[\{I^*, K^*\}\frac{dT_e'}{dx'} + \{J^*, L^*\} e E_{nl}'\right] \end{aligned} \tag{11}$$

where $eE_{nl} \equiv eE + T_e d\ln n/dx - 5/2 dT_e/dx$, and the kernels I^*, J^*, K^*, and L^* are functions of $\theta \equiv |\xi - \xi'|/T'^2$ given in terms of one single integral,

$$J^*(\theta) = 8\pi^{1/2} \int_0^\infty s^{3/2} \exp(-s - \theta/s^{3/2}) ds,$$

$I^* = 3J^* - 2\theta dJ^*/d\theta$, $L^* = (3I^* + J^*)/4$, $K^* = 4L^* - 2\theta dL^*/d\theta$. For H large, only the complete integrals of the kernels are needed; for small H, only the values at $\theta = 0$ are needed. It can also be shown that

$$\frac{\int_0^\infty K^* d\theta}{\int_0^\infty L^* d\theta} - \frac{\int_0^\infty I^* d\theta}{\int_0^\infty J^* d\theta} = \frac{K^*(0)}{L^*(0)} - \frac{I^*(0)}{J^*(0)} = 1.$$

Consequently, the formula[2]

$$q_T = \int \frac{-n' T_e' dx'}{4\pi(3m_e Z T_e')^{1/2}}\ L^* \frac{dT_e'}{dx}$$

agrees exactly with (11) for both long (or classical) and short scale limit profiles, and it should be a convenient approximation for intermediate cases. Luciani and co-workers first developed, in a series of papers, [5] a nonlocal formalism for the heat flux, using a fit to numerical results. Lindman and Swartz have given an alternative convolution formula.[6] Holstein and Decoster[7] discussed and compared nonlocal models. An extension of results to the $Z = 0(1)$ case was given in Ref.8, and applied to the structure of a plasma shock in Ref.9.

REFERENCES

[1] Albritton, J.R., Williams, E.A., Bernstein, I.B., and Swartz, K.P., *Phys.Rev. Lett.* 57, 1887 (1986).

[2] Sanmartín, J R , Ramírez, J , and Fernandez-Feria, R , *Phys Fluids* B 2, 2519 (1996)

[3] Prassad M K and Kershaw, D S , *Phys Fluids B* 1, 2430 (1989)

[4] Ramírez, J , "Conducción Electrónica no Clásica en Plasmas Producidos por Luz Laser", Ph D Thesis, Universidad Politécnica de Madrid (1990)

[5] Luciani, J F , Mora, P , and Virmot, J , *Phys Rev Lett* 51, 1664 (1983)

[6] Lindman, E L and Swartz, K , *Phys Fluids* 29, 2657 (1986)

[7] Holstein, P A and Decoster, A , *J Appl Phys* 62, 3592 (1987)

[8] Sanmartín, J R , Ramírez, J , Fernández-Feria, R , and Minotti, F , *Phys Fluids B* 4, 3579 (1992)

[9] Ramírez, J , Fernández-Feria, R , and Sanmartín, J R , *Phys Fluids B* 5, 1485 (1993)